age; height when alive, 4 feet 11½ inches; interred on right side; in front of the girl were the fragmentary bones of a child under 5 years of age. Arranged round the body of the girl—one of the Bronze Age dolichocephali—were 158 chalky fossil Echini (86 of these were perfect, the remainder were broken examples), together with a stone muller, several scrapers, a few chevron-marked fragments of a British pot, and other articles.

The other skeleton is that of a boy or girl—one of the brachycephali—interred on the left side with knees drawn towards chin; age about 14 years; height when alive, 3 feet 7½ inches. A few flint flakes, and many fragments of British pottery were in the grave. Near the right hand was a nodule of iron pyrites.

Numerous bone fragments belonging to *Bos longifrons* were in the covering material of both the latter graves.

I have a femur, humerus, and a few other bones of a fourth Celt from the floor of a British hut on Blow's Downs, Dunstable; several flakes and a scraper were with the skeleton, together with a block of pyrites and a metatarsal bone of a horse. The human femur shows the height of the living individual to have been 5 feet 10 inches. I was not in time to secure all this skeleton; more than one-half, including the skull, was shovelled with chalk into a lime-kiln.

Towards the end of last month (January), whilst opening four presumed Saxon (English) graves on a hill at Chalton, near here, I noticed that one of the extended skeletons had but one leg; the left leg had been entirely removed at the hip during life, or before burial. I could see no difference in the condition of the two acetabula. I took a small corroded iron knife from the front of two of the skeletons just where a strap would be bound round the waist. Three bodies were interred with the head to the west, the fourth with the head to the north. I have preserved the femora, humeri, and skull of the last mentioned; the skull exhibits every tooth sound and perfect in both jaws.

Dunstable. WORTHINGTON G. SMITH.

TIDAL PREDICTION.[1]

AT most places in the North Atlantic the prediction of high and low water is fairly easy, because there is hardly any diurnal tide. This abnormality makes it sufficient to have a table of the mean fortnightly inequality in the height and interval after lunar transit, supplemented by tables of corrections for the declinations and parallaxes of the disturbing bodies. But when there is a large diurnal inequality, as is commonly the case in other seas, the heights and intervals, after the upper and lower lunar transits, are widely different; the two halves of each lunation differ much in their characters, and the season of the year has great influence. Thus simple tables, such as are applicable in the absence of diurnal tide, are of no avail.

The tidal information supplied by the Admiralty for such places consists of rough means of the rise and interval at spring and neap, modified by the important warning that the tide is affected by diurnal inequality. Information of this kind affords scarcely any indication of the time and height of high and low water on any given day, and must be almost useless.

This is the present state of affairs at many ports of some importance, but at others a specially constructed tide-table for each day of each year is published in advance. A special tide-table is clearly the best sort of information for the sailor, but the heavy expense of prediction and publication is rarely incurred except at ports of first-rate commercial importance.

There is not any arithmetical method in use of computing a special tide-table which does not involve much work and expense. The admirable tide-predicting instrument of the Indian Government renders the prediction comparatively cheap; yet the instrument can hardly be deemed available for the whole world, and the cost of publication is so considerable that the instrument cannot, or at least will not, be used for many ports at remote places. It is not impossible, too, that national pride may deter the naval authorities of other nations from sending to London for their predictions, although the instrument may be used on the payment of certain fees.

The object, then, of the paper was to show how a general tide-table, applicable for all time, may be given in such a form that anyone with an elementary knowledge of the *Nautical Almanac* may, in a few minutes, compute two or three tides for the days on which they are required. The tables are also to be such that a special tide-table for any year may be computed with comparatively little trouble.

Any tide-table necessarily depends on the tidal constants of the particular port for which it is designed, and it is supposed in the paper that the constants are given in the harmonic system, and are derived from the reduction of tidal observations.

The complete expression for the height of water at any time consists of a number of terms, each of which involves some or all of the mean longitudes of moon, sun, lunar and solar perigees; there are also certain corrections, depending on the longitude of the moon's node. The variability of the height of water depends principally on the mean longitudes of the moon and the sun, and to a subordinate degree on the longitude of lunar perigee and node, for the solar perigee is sensibly fixed. There are, therefore, two principal variables, and two subordinate ones. This statement suggests the construction of a table of double entry for the variability of tide due to the principal variables, and of correctional tables for the subordinate ones; and this is the plan developed in the paper.

The tide-table finally consists of the interval after moon's transit, and height of high and low water, together with corrections depending on the longitude of the moon's node and on her parallax, computed for every 20m. of moon's transit, and for about every ten days in the year. Each table serves for the two times of year at which the sun's longitude differs by 180°, and they may be used without interpolation.

It was hoped that less elaborate tables might have sufficed, but it appeared that, at a station with very large diurnal inequality, the changes during the lunation, and with the time of year, in the interval and height are so abrupt and so great, that short tables would give very inaccurate results, unless used with elaborate interpolations. It is out of the question to suppose that a ship's captain would or could carry out these interpolations, and it is therefore proposed to throw the whole of that work on to the computer of the table.

Such a paper as this is only complete when an example has been worked out to test the accuracy of the tidal prediction, and when rules for the arithmetical processes have been drawn up, forming a complete code of instructions to the computer.

The port of Aden was chosen for the example because its tides are more complex and apparently irregular than those of any other place which has been thoroughly treated.

The arithmetic of the example was long, and was rearranged many times. An ordinary computer is said to work best when he is ignorant of the meaning of his work, but in this kind of tentative work a satisfactory arrangement cannot be attained without a full comprehension of the reason of the method. The author was therefore fortunate in securing the enthusiastic assistance of Mr. J. W. F. Allnutt, and expressed his warm thanks for the laborious computations carried out. After computing fully half the original table, Mr. Allnutt made a comparison for the whole of 1889 of predictions made by the new tables with those of the Indian Government. Without going into the details of this comparison, it may be mentioned that the probable error of the discrepancy between the two tables was 9m. in time, and 1·2 inches

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[1] Abstract of the Bakerian Lecture, delivered on January 29, by G. H. Darwin, F.R.S., Plumian Professor and Fellow of Trinity College, Cambridge.

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in the height of high water; that there were reasons to expect some systematic difference between the two calculations, and that all the considerable errors of time fall on those very small rises of water which are of frequent occurrence at Aden.

Two other comparisons were also made, one with the Indian predictions of 1887, and the other with actuality of 1884. In the latter case, when a few very small tides were omitted, the probable error was 7m. in the time, and 1·4 inches in height. It is concluded from these comparisons that, with good values for the tidal constants, the tables lead to excellent predictions, even better than are required for nautical purposes.

It is probable that this method may be applied to ports of second-rate importance, where there are not sufficient data for very accurate determination of the tidal constants. Suggestions are made for very large abridgment of the tables in such cases, accompanied, of course, by loss of accuracy.

The question of how far to go in each case must depend on a variety of circumstances. The most important consideration is likely to be the amount of money which can be expended on computation and printing; and after this will come the trustworthiness of the tidal constants, and the degree of desirability of an accurate tide-table. The aim of the paper was to give the tables in a simple form, and if, as seems certain, the mathematical capacity of an ordinary ship's captain will suffice for the use of the tables, whether in full or abridged, the principal object in view has been attained.

THEORY OF FUNCTIONS.[1]

I.

THE papers in Herr Schwarz's "Gesammelte Abhandlungen" range over a wide field. But varied as the subject-matter of the papers is, most of them have an internal connection which lies in profound study of the theory of functions.

Herr Schwarz is a pupil of Weierstrass, but he has been greatly influenced by the writings of Riemann. Both these eminent mathematicians have developed a theory of functions. Whilst Weierstrass starts in all his investigations with analytical expressions and operations, Riemann, following methods borrowed from the theory of potential, bases his theory on certain partial differential equations which express fundamental properties of functions, and by developing general properties of all functions tries to replace calculation by reasoning. Both applied their theories to the Abelian functions, and here Riemann's investigations proved to be more general. His speculations are, however, of such generality, that objections were raised by Kronecker and Weierstrass as to the validity of his reasoning, and thus it became doubtful whether his most important theorems were actually proved.

In consequence, various mathematicians have attempted to place Riemann's theorems on a more satisfactory basis, to graft Riemann's far-reaching speculations on the more strongly-rooted methods of Weierstrass. These attempts were made, by Neumann and others, in connection with the theory of potential, whilst other mathematicians used purely analytical investigations. Among the latter, Schwarz stands out most prominently, and those of his papers which relate to this subject seem to me to deserve most attention.

As the subject in its great generality and abstractness has received little attention in England, it seems desirable to give an outline of Riemann's theory in order to be the better able to appreciate Schwarz's papers. The notion of a function, originally equivalent to that of a power, was gradually extended till Bernoulli gave a first definition, which was made more concise by Leibnitz. According to them, y is called a function of x if there exists an equation between these variables which makes it possible to calculate y for any given values of x for all (real) values of x from $-\infty$ to $+\infty$. This last condition excludes series which are convergent only for certain values of x. The functions here included will be essentially those which are given by an equation involving a finite number of algebraical, trigonometrical, and logarithmic or exponential terms. Such a function will, as a rule, be continuous—practically the only exceptions being infinite values of y—and it will have a derived function. In fact, these two properties, continuity and existence of a derived function, were considered as identical. Such a function is geometrically represented by a curve, x and y being taken as Cartesian co-ordinates of a point. But not every curve can be taken as representing a function.

The next step forward is chiefly due to the work of Fourier. Physical problems required the study of functions with new characteristics. The initial temperature of a rod, lying in the axis of x, may be supposed arbitrarily given, not for all values of x, but only for those which lie within the interval covered by the rod, and in the trigonometrical series means were found for expressing this arbitrarily given temperature in terms of x. This led Dirichlet to the new definition: y is called a function of x if the values of y are *arbitrarily given*, say by a curve, for all values of *x within a given interval*. This involves a deviation from Bernoulli's definition in two directions: the function is originally not given by a mathematical expression, and not for all values of x; in fact, it exists only as far as it is given. The *expression*, by Fourier's series, is not wanted for the definition of the function; it is needed in order to make the latter amenable to mathematical treatment.

The study of functions thus defined leads to new ideas of possible discontinuities. The curve representing the function may have sharp bends, may even have sudden breaks, the value of y suddenly changing from one value to another, different by a finite quantity. At such points we cannot any longer speak of a derived function, though the expression by Fourier's series is still existing. Further investigations in this direction have clearly established the fact that the existence of a derived function is not at all a necessary consequence of the continuity of the function itself. In connection with this it may be mentioned that Riemann has given a first example of a function which is defined by a process of calculating y from x, such that for every value of x there exists a value of y which has within a finite interval an infinite number of discontinuities, and which therefore has no derived function, though it has an integral. Schwarz gives, in one of his papers, an example of a function which, though continuous, has no derived function.

A much greater revolution in the ideas about function was produced by the introduction of imaginary values of the variables. Cauchy defines, in conformity with the second of the above definitions, that $w = u + iv$ is a function of $z = x + iy$ if for each set of values xy the corresponding values of u and v are given. Here a geometrical representation of the variables becomes very useful, if not absolutely necessary, viz. the variable z is represented by that point z in a plane which has the rectangular co-ordinates x, y; and w by that point in another plane which has the rectangular co-ordinates u, v. This enables us to make use of geometrical conceptions, and thus to become more concise in our language. We shall speak of the value z and of the point z representing this value as equivalent, the one completely determining the other.

If we start with a given value z_0 of z, and vary the latter, then the point z will move from the position z_0,

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[1] "Gesammelte Abhandlungen." Von H. A. Schwarz. Two Volumes. Berlin: Julius Springer, 1890.)

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